Problem Review Session 7
PHYS 741

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Disclaimer: The problems below are not my own making but are taken from A Guide to Physics Problems: Part 2 (GPP2), Princeton Problems in Physics (PPP), and past qualifying exams from UNC (Qual).

Practice Problems

1. **(PPP 4.2)** Consider a heteronuclear diatomic molecule with moment of inertia I. In this problem, only the rotational motion of the molecule should be considered.
   (a) Using classical statistical mechanics, calculate the specific heat $C(T)$ of this system at temperature T.
   (b) In quantum mechanics, this system has energy levels
   $$E_j = \frac{\hbar^2}{2I} j(j+1) \qquad j = 0, 1, 2, \ldots$$
   Each j level is $(2j+1)$-fold degenerate. Using quantum statistical mechanics, find expressions for the partition function $\mathcal{Z}$ and the average energy $\langle E \rangle$ of this system, as a function of temperature. Do not attempt to evaluate these expressions.
   (c) By simplifying your expressions in (b), derive an expression for the specific heat $C(T)$ that is valid at very low temperatures. In what range of temperatures is your expression valid?
   (d) By simplifying your answer to (b), derive a high-temperature approximation to the specific heat $C(T)$. What is the range of validity of your approximation?

2. **(Qual 2015 SM-4)** We are interested in some basic properties of the density matrix in quantum statistical mechanics. Consider a system with Hamiltonian H. Let the set of normalized states $|\psi_k(t)\rangle$ be an ensemble of possible states of the system obeying the Schrodinger equation. The density matrix is given by $\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$.
   (a) Given that the trace of ρ^2 is equal to 1 for pure states and less than 1 for a mixture, show that a pure state cannot evolve into a mixture or vice versa by considering $\frac{\partial \mathrm{Tr}\rho^2}{\partial t}$.
   (b) Show that equilibrium statistical mechanics is described by a density matrix of the form $\rho = \rho(H)$ by considering $\frac{\partial \rho}{\partial t}$.

3. **(GPP2 4.47)** The upper end of a hanging chain is fixed while the lower end is attached to a mass M. The (massless) links of the chain are ellipses with either the major axis or the minor axis vertical. The major axis has a length of $l + a$, while the minor axis has a length of $l - a$. Assume that the chain has N links and is in thermal equilibrium at temperature T.
   (a) Find the partition function.
   (b) Find the average length of the chain.