Problem Review Session 5
PHYS 741

Zach Nasipak

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Disclaimer: The problems below are not my own making but are taken from Pathria's Statistical Mechanics (PSM) and past qualifying exams from UNC (Qual).

Practice Problems

1. **(Qual 2015 SM-1)** Consider an ideal gas in a one-dimensional channel of length L. The energy of the particle is given by $E = (p^2/2m) - \varepsilon_0$.

   (a) Show, using the classical approach, that the partition function of one particle is given by

   $$Q_1(T, L) = \frac{L}{\lambda} e^{\varepsilon_0/kT} \qquad \text{and} \qquad \lambda = \frac{h}{\sqrt{2\pi mkT}}$$

   (b) Calculate the chemical potential of this system of N indistinguishable particles at temperature T.

   Reminder: $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$.

2. **(Qual 2012 SM-5)** A cylinder of radius R and length L contains N molecules of mass m of an ideal gas at temperature T. The cylinder rotates about its axis with an angular velocity ω. Find a change in the free energy of the gas ΔA, as compared to that at rest.

3. **(PSM 3.31)** Study the statistical mechanics of a system (i.e. calculate the entropy, chemical potential, pressure, internal energy and heat capacities) of N "Fermi oscillators," which are characterized by only 2 eigenvalues, namely 0 and ε. (This is similar to Problem 6.3 in Huang only you are now doing it from canonical approach.)

Additional Problem

4. **(PSM 3.18)** Show that for a system in the canonical ensemble

   $$\langle (\Delta E)^3 \rangle = k^2 \left[T^4 \left(\frac{\partial C_V}{\partial T} \right)_V + 2T^3 C_V \right].$$

   Verify that for an ideal gas

   $$\left\langle \left(\frac{\Delta E}{U} \right)^2 \right\rangle = \frac{2}{3N} \qquad \text{and} \qquad \left\langle \left(\frac{\Delta E}{U} \right)^3 \right\rangle = \frac{8}{9N^2}.$$

5. **(PSM 3.26)** The energy eigenvalues of an s-dimensional harmonic oscillator can be written as $\varepsilon_j = (j + s/2)\hbar\omega$.

Show that the jth energy level has a multiplicity of $(j+s-1)!/j!(s-1)!$. Evaluate the partition function, and the major thermodynamic properties of a system of N such oscillators, and compare your results with a corresponding system of sN one-dimensional oscillators. Show in particular that the chemical potential $\mu_s = s\mu_1$.