Problem Review Session 4
PHYS 741

Zach Nasipak

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Disclaimer: The problems below are not my own making but are taken from Pathria's Statistical Mechanics (PSM).

Practice Problems

1. **(PSM 1.8)** Consider a system of quasiparticles whose energy eigenvalues are given by

$$\varepsilon = nh\nu; \qquad n = 0, 1, 2, \ldots .$$

Obtain an asymptotic expression ($N \gg 1$, $E/N \gg 1$) for the number of microstates Ω of this system for given number N of quasiparticles and a given total energy E. Determine the temperature T of the system as a function of E/N and $h\nu$, and examine the situation for which $E/Nh\nu \gg 1$.

2. **(PSM 2.7)**
   (a) Derive an asymptotic expression ($N \gg 1$, $E/N \gg 1$) for the number of ways in which a given energy E can be distributed among a set of N one-dimensional harmonic oscillators, with the energy eigenvalues of the oscillators being $(n + 1/2)\hbar\omega; n = 0, 1, 2, \ldots$.
   (b) Derive the corresponding expression for the "volume" of the relevant region of phase space of this system.
   (c) Establish the correspondence between the two results of (a) and (b), showing that the conversion factor ω_0 is precisely h^N. (For those reading Huang's Statistical Mechanics, ω_0 essentially refers to the volume in phase space occupied by a single microstate.)

3. **(PSM 2.8)** Show that

$$V_{3N} = \int \cdots \int_{0 \le \sum_{i=1}^{N} r_i \le R} \prod_{i=1}^{N} \left(4\pi r_i^2 dr_i\right) = \frac{(8\pi R^3)^N}{(3N)!},$$

where V_{3N} is the volume of a $3N$-dimensional hypersphere of radius R. Using this results, compute the "volume" of the relevant region of the phase space of an extreme relativistic gas ($\varepsilon = pc$) of N particles moving in three-dimensions. Hence, derive expressions for the various thermodynamic properties of this system (energy, entropy, chemical potential, equation of state, and $\gamma = C_P/C_V$).

Hint: Begin with the definition of the n-dimensional hypersphere volume

$$V_n = \int \cdots \int_{0 \le \sum_{i=1}^{n} x_i^2 \le R^2} \prod_{i=1}^{n} \left(dx_i\right),$$

to find the integral form of V_{3N}. Then evaluate the integral by using the fact that $V_{3N} = C_{3N} R^{3N}$, where C_{3N} is a constant of proportionality and use the integral

$$\int_0^\infty e^{-r} r^2 dr = 2,$$

to solve for C_{3N}.

Additional Problem

If you want to try another problem similar to PSM 2.8

1. **(PSM 2.9)** Solve the integral

$$\underset{0 \leq \sum_{i=1}^{3N} |x_i| \leq R}{\int \cdots \int} (dx_1 dx_2 \cdots dx_{3N}),$$

and use it to determine the "volume" of the relevant region of the phase space of an extreme relativistic gas ($\varepsilon = pc$) of $3N$ particles moving in one-dimension. Determine, as well, the number of ways of distributing a given energy E among this system of particles and show that (asymptotically) $\omega_0 = h^{3N}$.

Pathria Notation

Useful Pathria notation

- $\Omega = \Omega(N, E, V)$ refers to the number of microstates that have energy E, the number of particles N, and occupy a volume V.
- $\Gamma = \Gamma(N, E, V; \Delta)$ refers to the number of microstates that have energy $E \leq E' \leq E + \Delta$, the number of particles N, and occupy a volume V.
- ω refers to volume of phase space confined to the region $E \leq H(p_i, q_i) \leq E + \Delta$. Huang refers to this as $\Gamma(E)$. A useful relation is that $\Gamma = \omega/\omega_0$, where ω_0 is described in the problem above.
- Σ refers to the volume of phase space confined to the region $E \leq H(p_i, q_i)$, just like in Huang.
- $g(x)$ refers to the density of states of a variable x. x can refer to energy, momentum, position, etc. Therefore $g(E)$ in Pathria is essentially the same as $\omega(E)$ in Huang.