Problem Review Session 3
PHYS 741

Zach Nasipak

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Disclaimer: The problems below are not my own making but are taken from Princeton Problems in Physics (PPP) and past qualifying exams from UNC (Qual).

Past Qualifying Exam Problems

1. **(Qual 2015 SM-2)** Consider a white dwarf star where the number of electrons is N, the mass of the star is $M = 2Nm_p$ (where m_p is the mass of the proton), and the volume of the star is V. The pressure of an ideal Fermi gas is given by

$$P = \frac{8\pi}{3h^3} \int_0^\infty \frac{1}{e^{(\epsilon-\mu)/kT} + 1} \left(p \frac{\partial \epsilon}{\partial p} \right) p^2 dp,$$

where μ is the chemical potential and ϵ is the relativistic kinetic energy given by

$$\epsilon = m_e c^2 \left\{ \left[1 + \left(\frac{p}{m_e c} \right)^2 \right]^{1/2} - 1 \right\},$$

where m_e is the mass of the electron and c is the speed of light. It can be shown that the Fermi momentum is given by $p_F = \frac{3N}{8\pi V}^{1/3} h$, where h is the Planck constant. Show that in the $T \to 0$ limit, the radius of the star R is given by the equation

$$\frac{8\pi m_e^4 c^5}{3h^3} \int_0^{\theta_F} \sinh^4 \theta d\theta = \frac{\alpha}{4\pi} \frac{GM^2}{R^4}, \qquad \text{where} \qquad m_e c \sinh \theta_F = p_F.$$

Here $\alpha \simeq 1$ is a known constant, and G is the gravitational constant.

2. **(Qual 2014 SM-1)** Consider a system of N classical distinguishable harmonic oscillators where the Hamiltonian is given by

$$H = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 q_i^2 \right).$$

(a) Calculate $\Sigma(N, E)$, the total number of microstates with energy less than or equal to E.

(b) Based on the calculated $\Sigma(N, E)$, show that the entropy is given by

$$S(N, E) = Nk \left[1 + \ln \left(\frac{E}{N\hbar\omega} \right) \right].$$

Practice Problems

3. **(PPP 4.1)** Consider a system of $N \gg 1$ non-interacting particles in which the energy of each particle can assume two and only two distinct values: 0 and E ($E > 0$). Denote by n_0 and n_1 the occupation numbers of the energy levels 0 and E, respectively. The fixed total energy of the system is U.
   (a) Find the entropy of the system
   (b) Find the temperature as a function of U. For what range of values of n_0 is $T < 0$?
   (c) In which direction does heat flow when a system of negative temperature is brought into thermal contact with a system of positive temperature? Why?

4. **(PPP 4.7)** A wire of length l and mass per unit length μ is fixed at both ends and tightened to a tension τ. What is the root mean square fluctuation, in classical statistics, of the midpoint of the wire when it is in equilibrium with a heat bath at temperature T? A useful series is

$$\sum_{m=0}^{\infty} (2m+1)^{-2} = \frac{\pi^2}{8}.$$