Problem Review Session 2
PHYS 741

Zach Nasipak

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Disclaimer: The problems below are not my own making but are taken from A Guide to Physics Problems: Part 2 (GPP2).

Practice Problems

1. **(GPP2 4.11 Liquid-Solid-Liquid)** A small amount of water of mass $m = 50$ g in a container at temperature $T = 273$ K is placed inside a vacuum chamber which is evacuated rapidly. As a result, part of the water freezes and becomes ice and the rest becomes vapor.
   (a) What amount of water initially transforms into ice? The latent heat of fusion (ice to water) $q_i = 80$ cal/g, and the latent heat of vaporization (water to vapor) $q_v = 600$ cal/g.
   (b) A piece of heated metal alloy of mass $M = 325$ g and original volume $V = 48$ cm^3 is placed inside the calorimeter together with the ice obtained as a result of the experiment in (a). The density of metal at $T = 273$ K is $\rho_0 = 6.8$ g/cm^3. The thermal capacity is $C = 0.12$ cal/g K, and the coefficient of linear expansion $\alpha = 1.1 \times 10^{-5}$ K^{-1}. How much ice will have melted when equilibrium is reached?

2. **(GPP2 4.20 Adiabatic Atmosphere)** The lower $10 - 15$ km of the atmosphere, the troposphere, is often in a convective steady state with constant entropy, not constant temperature. (PV^γ is independent of the altitude, where $\gamma \equiv C_P/C_V$.)
   (a) Find the change of temperature in this model with altitude dT/dz.
   (b) Estimate dT/dz in K/km. Consider the average diatomic molecule of air with molar mass $\mu = 29$ g/mol.

3. **(GPP2 4.29 Heat Extraction)**
   (a) A body of mass M has a temperature-independent specific heat C. If the body is heated reversibly from a temperature T_i to a temperature T_f, what is the change in its entropy?
   (b) Two such bodies are initially at temperatures of 100 K and 400 K. A reversible engine is used to extract heat with the hotter body as a source and the cooler body as a sink. What is the maximum amount of heat the can be extracted in units of MC?
   (c) The specific heat of water is $C = 4.2$ J/g K, and its density is 1 g/cm^3. Calculate the maximum useful work that can be extracted, using as a source 10^3 m^3 of water at 100°C and a lake of temperature 10°C as a sink.

4. **(GPP2 4.13 Maxwell Boltzmann Averages)**
   (a) Write the properly normalized Maxwell-Boltzmann distribution $f(v)$ for finding particles of mass m with magnitude of velocity in the interval $[v, v + dv]$ at a temperature T. (Hint: $f(E) \sim e^{-E/kT}$.)
   (b) What is the most likely speed at temperature T?
   (c) What is the average speed?

(d) What is the average square speed?

5. **(GPP2 4.35 Poisson Distribution in Ideal Gases)** Consider a monatomic ideal gas of total N' molecules in a volume V'. Show that the probability P_N for the number N of molecules contained in a small element of V is given by the Poisson distribution

$$P_N = \frac{e^{-\langle N \rangle} \langle N \rangle^N}{N!}$$

Another Rocket Problem

1. **(GPP2 4.19 Rocket in Drag)** A rocket has an effective frontal area A and blasts off with a constant acceleration a straight up from the surface of the Earth.

   (a) Use either dimensional analysis or an elementary derivation to find out how the atmospheric drag on the rocket should vary as some power(s) of the area A, the rocket velocity v, and the atmospheric density ρ (assuming that we are in the region of high Reynolds numbers).

   (b) Assume that the atmosphere is isothermal with temperature T. Derive the variation of the atmospheric density ρ with height z. Assume that the gravitational acceleration g is a constant and that the density at sea level is ρ_0.

   (c) Find the height h_0 at which the drag on the rocket is at a maximum.