Problem Review Session 1
PHYS 741

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Disclaimer: The problems below are not my own making but are taken from A Guide to Physics Problems: Part 2 (GPP2) and Princeton Problems in Physics (PPP).

Practice Problems

1. **(GPP2 4.12 Hydrogen Rocket)** The reaction chamber of a rocket engine is supplied with a mass flow rate m of hydrogen and sufficient oxygen to allow complete burning of the fuel. The cross section of the chamber A, and the pressure at the cross section is P with temperature T. Calculate the force that the chamber is able to provide.

2. (**PPP 4.4 Phase Coexistence**) The temperature of a long vertical column of a particular substance is T everywhere. Below a certain height $h(T)$ the substance is solid, whereas above $h(T)$ it is in a liquid phase. Calculate the density difference $\Delta\rho = \rho_s - \rho_l$ between the solid and liquid ($|\Delta\rho| \ll \rho_s$) in terms of L (the latent heat of fusion per unit mass), dh/dT, T, ρ_l and g, the acceleration due to gravity.

3. **(GPP2 4.51 Nonideal Gas Equation)** A gas obeys the equation of state

   $$P = Nk\left(\frac{T}{V} + \frac{B(T)}{V^2}\right)$$

   where B(T) is a function of the temperature T only. The gas is initially at temperature T and volume V_0 and is expanded isothermally and reversibly to volume $V_1 = 2V_0$.

   (a) Find the work done in the expansion.

   (b) Find the heat absorbed in the expansion.

4. **(PPP 4.5 Otto Cycle)** The operation of a gasoline engine is (roughly) similar to the Otto cycle (Fig 1), where:

   - $A \to B$: Gas compressed adiabatically
   - $B \to C$: Gas heated isochorically (constant volume; corresponds to combustion of gasoline)
   - $C \to D$: Gas expanded adiabatically (power stroke)
   - $D \to A$: Gas cooled isochorically

   Compute the efficiency of the Otto cycle for an ideal gas (with temperature-independent heat capacities) as a function of the compression ratio V_A/V_B, and the heat capacity per particle C_V.

5. **(GPP2 4.32 Joule Cycle)** Find the efficiency of the Joule cycle, consisting of two adiabats and two isobars (see Fig 2). Assume that the heat capacities of the gas C_P and C_V are constant.

6. **(GPP2 4.53 Critical Parameters)** Consider a system described by the Dietrici equation of state

   $$P(V - nB) = nN_AkTe^{-nA/N_AkTV}$$

   where A, B, k, N_A and R are constants and P, V, T, and n are the pressure, volume, temperature, and number of moles. Calculate the critical parameters, i.e., the values of P, V, and T at the critical point.

Additional Problems

1. **(GPP2 4.56 Osmotic Pressure)** Consider an ideal mixture of N_0 monatomic molecules of type A and N_2 monatomic molecules of type B in a volume V.

   - Calculate the free energy $F(T, V, N_0, N_1)$. Calculate the Gibbs potential $G(T, P, N_0, N_1)$. Notice that G is the Legendre transform of F with respect to V.
   - If $N_0 \gg N_1$, the molecules of type A are called the solvent, and those of type B the solute. Consider two solutions with the same solvent (type A) and different concentrations of solute (type B) separated by a partition through which solvent molecules can pass but solute molecules cannot. There are N_0 particles in volume V (or $2N_0$ in volume $2V$), and N_1 and N_2 particles in volume V on the left and right of the membrane, respectively. Calculate the pressure difference across the membrane at a given temperature and volume. Assume the concentrations of the solutions are small; i.e.

     $$C_1 = \frac{N_1}{N_0} \ll 1$$

     and

     $$C_2 = \frac{N_2}{N_0} \ll 1$$

2. **(GPP2 4.32 Teacup Engine)** An astronaut is peacefully drinking a cup of tea, when she had an emergency outside the shuttle, and had to do an EVA to deal with it. Upon leaving the shuttle, her jetpack failed, and nothing remained to connect her to the shuttle. Fortunately, she had absentmindedly brought her teacup with her. Since this was the only teacup she had, she did not want to throw it away in order to propel her back to the shuttle. Instead, she used the sublimation of the frozen tea to propel her back to the spaceship. Was it really possible? Estimate the time it might take her to return if she is a distance $L = 40$ m from the shuttle. Assume the sublimation occurs at a constant temparture $T = 273$ K.

3. **(GPP2 4.10 Heat Loss)** An immersion heater of power $J = 500$ W is used to heat water in a bowl. After 2 minutes, the temperature increases from $T_1 = 85°$C to $T_2 = 90°$C. The heater is then switched off for an additional minute, and the temperature drops by $\Delta T = 1°$C. Estimate the mass m of the water in the bowl. The thermal capacity of water $c = 4.2 \times 10^3$ J/kg K.